Life Insurance Mathematics
Types of Insurance
Q&A

Whole Life

Discrete

$$\boldsymbol{Z} = v^{K_x+1}$$

$$E[\boldsymbol{Z}] = A_x = \sum_{k=0}^{oo} v^{k+1} \ {}_kp_x \, q_{x+k}$$

Continuous

$$\boldsymbol{Z} = e^{-\delta T_x}$$

$$E[\boldsymbol{Z}] = \bar{A}_x = \int_0^{oo} e^{-\delta t} \ {}_tp_x \mu_{x+t} dt$$

$$Var[\boldsymbol{Z}] = {}^2\bar{A}_x - (\bar{A}_x)^2$$

Term Insurance:

Discrete

$$\boldsymbol{Z} = \begin{cases} v^{K_x+1} & K_x < n \\ 0 & K_x \geq n \end{cases}$$

$$E[\boldsymbol{Z}] = A^1_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1} \ {}_kp_x \, q_{x+k}$$

Continuous

$$\boldsymbol{Z} = \begin{cases} e^{-\delta T_x} & T_x < n \\ 0 & T_x \geq n \end{cases}$$

$$E[\boldsymbol{Z}] = \bar{A}^1_{x:\overline{n|}} = \int_0^{n} e^{-\delta t} \ {}_tp_x \mu_{x+t} dt$$

$$Var[\boldsymbol{Z}] = {}^2\bar{A}^1_{x:\overline{n|}} - \left(\bar{A}^1_{x:\overline{n|}}\right)^2$$

Pure Endowment:

Discrete/Continuous

$$\boldsymbol{Z} = \begin{cases} 0 & K_x < n \\ v^n & K_x \geq n \end{cases}$$

$$E[\boldsymbol{Z}] = A_{x:\overset{\;\;1}{\overline{n|}}} = {}_nE_x = v^n \, {}_np_x$$

$$E[\boldsymbol{Z^2}] = {}^2A_{x:\overset{\;\;1}{\overline{n|}}} = v^{2n} \, {}_np_x$$

Q1. For a whole life insurance on (40), you are given

i. Death benefit $100,000 is paid at the moment of death.

ii.

$$\mu_{40+t} = \begin{cases} 0.04, & 0 \le t \le 20 \\ 0.01, & 20 < t \end{cases} \qquad \delta_{40+t} = \begin{cases} 0.05, & 0 \le t \le 20 \\ 0.03, & 20 < t \end{cases}$$

Find the net single premium.

Solution:

$$\bar{A}_x = \bar{A}^1_{x:\overline{n|}} + {}_nE_x \cdot \bar{A}_{x+n}$$

$$\bar{A}_{40} = \bar{A}^1_{40:\overline{20|}} + {}_{20}E_{40} \cdot \bar{A}_{60}$$

$$\bar{A}_{40} = \int_0^{20} e^{-0.05t} e^{-0.04t} 0.04dt + e^{-0.05 \cdot 20} \cdot e^{-0.04 \cdot 20} \int_0^{oo} e^{-0.03t} e^{-0.01t} 0.01dt$$

$$\bar{A}_{40} = 0.04 \cdot \frac{e^{-0.09t}}{-0.09}\bigg]_0^{20} + e^{-20 \cdot (0.09)} \cdot \frac{0.01}{0.04} = 0.4123$$

Net single premium: 100,000 x 0.4123 = $41,230

Q2. Ölüm anında $\boldsymbol{K}$ teminatı veren bir tam hayat sigortasının bugünkü değeri $\boldsymbol{Z}$ rassal değişkeni ile gösterilmektedir. Mortality and continuous rate are given as 0.04 and 0.06 respectively. ve bu sigortanın net tek priminin $Var(\boldsymbol{Z})$ değerine eşit olduğu bilinmektedir. Find the value of $\boldsymbol{K}$.

Solution:

$$E(Z) = Var(Z) = E(Z^2) - [E(Z)]^2$$

$$K\frac{\mu}{\mu+\delta} = K^2\frac{\mu}{\mu+2\delta} - \left(K\frac{\mu}{\mu+\delta}\right)^2$$

$$K\frac{0.04}{0.04+0.06} = K^2\frac{0.04}{0.04+0.12} - \left(K\frac{0.04}{0.04+0.06}\right)^2$$

$$\boldsymbol{K} = 4.44$$

S3. Let X by the PV of a \$1, paid at the moment of death, n-year endowment and let Y be PV of a \$1, paid at the moment of death, n-year term insurance.

i. $Var(X) = 0.0078$
ii. $v^n = 0.3$
iii. ${}_np_x = 0.8$
iv. $E(Y) = 0.02$

$Var(Y) = ?$

Solution:

$$E[X] = \bar{A}_{x:\overline{n|}} = \bar{A}^1_{x:\overline{n|}} + {}_nE_x \quad (1)$$

$$Var(X) = {}^2\bar{A}_{x:\overline{n|}} - \left(\bar{A}_{x:\overline{n|}}\right)^2 = 0.0078 \quad (2)$$

$$E[Y] = \bar{A}^1_{x:\overline{n|}} = 0.02 \quad (3)$$

By using (1) and (3), equation (2) is:

$$Var(X) = {}^2\bar{A}_{x:\overline{n|}} - \left(\bar{A}^1_{x:\overline{n|}} + {}_nE_x\right)^2 = 0.0078$$

$$Var(X) = {}^2\bar{A}_{x:\overline{n|}} - \left(\bar{A}^1_{x:\overline{n|}} + v^n\, {}_np_x\right)^2 = 0.0078$$

$${}^2\bar{A}_{x:\overline{n|}} - (0.02 + 0.3 \cdot 0.8)^2 = 0.0078$$

$${}^2\bar{A}_{x:\overline{n|}} = 0.0754.$$

Recursive relation gives ${}^2\bar{A}^1_{x:\overline{n|}}$.

$${}^2\bar{A}_{x:\overline{n|}} = {}^2\bar{A}^1_{x:\overline{n|}} + v^{2n}\, {}_np_x$$

$$0.0754 = {}^2\bar{A}^1_{x:\overline{n|}} + 0.3^2 0.8$$

$${}^2\bar{A}^1_{x:\overline{n|}} = 0.0034$$

Then $Var(Y) = {}^2\bar{A}^1_{x:\overline{n|}} - \left(\bar{A}^1_{x:\overline{n|}}\right)^2 = 0.0034 - 0.02^2 = 0.003.$