

Retirement Gains:

Example:

Plan Provisions:

Retirement benefit:	2% of final earnings per year of credited service
Normal form of payment:	Life Only, payable monthly in advance
Normal retirement age:	Age 65
Unreduced retirement age:	Age 62
Early retirement reduction:	4% for each year prior to unreduced retirement age
Termination benefit:	Deferred pension payable at age 65, or actuarial equivalent if received earlier
Pre-retirement death benefit:	Actuarial present value of deferred pension payable from when the member would have attained age 65

Actuarial Assumptions and Methods:

Interest rate:	5% per year
Salary increase rate:	3% per year
Actuarial cost method:	Projected Unit Credit, prorated on service
Pre-retirement mortality:	None
Termination rates:	10% at age 50
Retirement rates:	See table below
Timing of decrements:	Beginning of year

Retirement Rates:

Age	Retirement
55	25%
62	50%
65	100%

Annuity Factors:

$\ddot{a}_{50}^{(12)}$	16.7
$\ddot{a}_{55}^{(12)}$	15.8
$\ddot{a}_{62}^{(12)}$	14.2
$\ddot{a}_{65}^{(12)}$	13.3

Member Data as at December 31, 2017:

	Member A	Member B
Age	50	61
Earnings for 2017	\$80,000	\$100,000
Credited Service	10 years	14 years

- (a) Calculate the accrued liability and normal cost at December 31, 2017 for each member.

Show all work.

You are given:

- Member A receives a salary increase of 7% at December 31, 2018.
- Member B died on December 31, 2018. As of December 31, 2018, the death benefit has not been paid.

- (b) Calculate the accrued liability at December 31, 2018 for each member.

Show all work.

- (c) Calculate the gains and losses by source for 2018.

Solution:

(a)

Member A

AL at Dec. 31, 2017 = AL (term 50) + AL (ret 55) + AL (ret 62) + AL (ret 65) =

\$162,135

AL (term 50) = $0.10 \times 2\% \times \$80,000 \times 10 \times 13.3 \times (1.05)^{-(65-50)} = \$10,236$

AL (ret 55) = $0.90 \times 0.25 \times 2\% \times \$80,000 \times (1.03)^{(55-50)} \times 10 \times 15.8 \times (1.05)^{-(55-50)}$
 $\times [1 - 0.04 \times (62 - 55)] = \$37,199$

AL (ret 62) = $0.90 \times 0.75 \times 0.5 \times 2\% \times \$80,000 \times (1.03)^{(62-50)} \times 10 \times 14.2 \times (1.05)^{-(62-50)}$
 $= \$60,878$

AL (ret 65) = $0.90 \times 0.75 \times 0.5 \times 2\% \times \$80,000 \times (1.03)^{(65-50)} \times 10 \times 13.3 \times (1.05)^{-(65-50)}$
 $= \$53,823$

NC at Dec. 31, 2017 = NC (term 50) + NC (ret 55) + NC (ret 62) + NC (ret 65) =

\$15,190

NC (term 50) = \$0

NC (ret 55) = $0.90 \times 0.25 \times 2\% \times \$80,000 \times (1.03)^{(55-50)} \times 1 \times 15.8 \times (1.05)^{-(55-50)}$
 $\times [1 - 0.04 \times (62 - 55)] = \$3,720$

NC (ret 62) = $0.90 \times 0.75 \times 0.5 \times 2\% \times \$80,000 \times (1.03)^{(62-50)} \times 1 \times 14.2 \times (1.05)^{-(62-50)}$
 $= \$6,088$

NC (ret 65) = $0.90 \times 0.75 \times 0.5 \times 2\% \times \$80,000 \times (1.03)^{(65-50)} \times 1 \times 13.3 \times (1.05)^{-(65-50)}$
 $= \$5,382$

Member B

AL at Dec. 31, 2017 = AL (ret 62) + AL (ret 65) = **\$367,427**

AL (ret 62) = $0.5 \times 2\% \times (1.03)^{(62-61)} \times \$100,000 \times 14 \times 14.2 \times (1.05)^{-(62-61)} = \$195,013$

AL (ret 65) = $0.5 \times 2\% \times \$100,000 \times (1.03)^{(65-61)} \times 14 \times 13.3 \times (1.05)^{-(65-61)} = \$172,414$

NC at Dec. 31, 2017 = NC (ret 62) + NC (ret 65) = **\$26,245**

NC (ret 62) = $0.5 \times 2\% \times (1.03)^{(62-61)} \times \$100,000 \times 1 \times 14.2 \times (1.05)^{-(62-61)} = \$13,930$

NC (ret 65) = $0.5 \times 2\% \times \$100,000 \times (1.03)^{(65-61)} \times 1 \times 13.3 \times (1.05)^{-(65-61)} = \$12,315$

Alternate solution: Since there is no assumed decrement in the beginning of the first year, then use the formula AL at Dec. 31, 2017 / Credited Service at Dec. 31, 2017 =

$\$367,427 / 14 = \mathbf{\$26,245}$

(b)

Member A (Age 51)

$$\text{Earning 2018} = \$80,000 * 1.07 = \$85,600$$

$$\text{AL at Dec. 31, 2018} = \text{AL (ret 55)} + \text{AL (ret 62)} + \text{AL (ret 65)} = \mathbf{\$202,508}$$

$$\text{AL (ret 55)} = 0.25 \times 2\% \times \$85,600 \times (1.03)^{(55-51)} \times 11 \times 15.8 \times (1.05)^{-(55-51)} \times [1 - 0.04 \times (62-55)] = \$49,593$$

$$\text{AL (ret 62)} = 0.75 \times 0.5 \times 2\% \times \$85,600 \times (1.03)^{(62-51)} \times 11 \times 14.2 \times (1.05)^{-(62-51)} = \$81,160$$

$$\text{AL (ret 65)} = 0.75 \times 0.5 \times 2\% \times \$85,600 \times (1.03)^{(65-51)} \times 11 \times 13.3 \times (1.05)^{-(65-51)} = \$71,755$$

Member B (Age 62)

$$\text{Earning 2018} = \$100,000 * 1.03 = 103,000$$

$$\text{AL at Dec. 31, 2018 (death benefit)} = 2\% \times \$103,000 \times 15 \times 13.3 \times (1.05)^{-(65-62)} = \$355,011$$