(1) $\int_0^2 x^3\sqrt{16-x^4}\,dx$ $\quad u = 16 - x^4$

$du = -4x^3dx$

$-\frac{1}{4}du = x^3dx$

$x=0 \longrightarrow u=16$

$x=2 \longrightarrow u=16-2^4=0$

$$\int_{16}^{0} -\frac{1}{4}\sqrt{u}\,du = \frac{1}{4}\int_0^{16} u^{1/2}du = \frac{1}{4}\frac{u^{3/2}}{3/2}\Bigg|_0^{16}$$

$$= \left(\frac{1}{4}\cdot\frac{2}{3}\right)\left(16^{3/2}-0\right) = \frac{1}{6}(64) = \frac{32}{3}$$

(2) $\begin{vmatrix} 1 & 1 & 2 \\ 3 & -2 & a \\ 2 & 1 & 2 \end{vmatrix} = 8 \qquad a=?$

$$1\cdot\begin{vmatrix} -2 & a \\ 1 & 2\end{vmatrix} - 1\begin{vmatrix} 3 & a \\ 2 & 2\end{vmatrix} + 2\begin{vmatrix} 3 & -2 \\ 2 & 1\end{vmatrix} = 8$$

$$(-4-a) - (6-2a) + 2(3+4) = 8$$

$$-4 - a - \cancel{6} + 2a + \cancel{6} + \cancel{8} = \cancel{8}$$

$$-4 + a = 0 \qquad \boxed{a=+4}$$

(3) $\int_1^e \ln(x)dx \qquad \int u\,dv = vu - \int v\,du$

$u = \ln x \qquad du = \frac{1}{x}dx$

$dv = dx \qquad v = x$

$$= x\ln x\Big|_1^e - \int x\,\frac{1}{x}dx = e\,\ln e - \cancel{1\ln 1}$$

$$-\int_1^e 1\cdot dx$$

$$= e - x\Big|_1^e = e - e + 1 = 1$$

④ $A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 5 & 3 & 4 \end{pmatrix}$ $f(x) = \det\left(\underline{A - xI}\right)$

$$f(x) = \det\left(\begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 5 & 3 & 4 \end{pmatrix} - \begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} 1-x & 0 & 0 \\ -2 & 3-x & 0 \\ 5 & 3 & 4-x \end{pmatrix} = (1-x)(3-x)(4-x)$$

kökler $\boxed{1, 3, 4}$

⑦ $\lim\limits_{x \to 0} \dfrac{\sin(1-e^{-x})}{x}$ $\to 0$

$$= \lim_{x \to 0} \boxed{\frac{\sin(1-e^{-x})}{1-e^{-x}}} \cdot \boxed{\frac{1-e^{-x}}{x}}$$

$\searrow 0$

$$\boxed{\lim_{u \to 0} \frac{\sin(u)}{u} = 1}$$

1 $= \lim\limits_{x \to 0} \dfrac{1-e^{-x}}{x}$ $\to 0$, $\searrow 0$

$$= \lim_{x \to 0} \frac{0-(-e^{-x})}{1} = \frac{1}{1} = 1.$$

L'Hospital

⑧ $\underline{f'(x) \le 12}$ $f(6) = -10$ $f(15) = ?$

$$\underline{\int_6^{15} f'(x)\,dx} \le \int_6^{15} 12 \cdot dx$$

$$f(15) - \underline{f(6)} \le 12x \Big|_6^{15} = 12(15-6) = 108$$

$$f(15) \le 108 + f(6) = 98$$

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$$f'(x) = x^3 - \frac{2}{x^2} + 2 \qquad f(1)=5 \qquad f(4)=?$$

$$f(x) = \int \left(x^3 - 2x^{-2} + 2\right)dx$$

$$f(x) = \frac{x^4}{4} - 2\,\frac{x^{-1}}{(-1)} + 2x + C$$

$$f(1) = \frac{1}{4} + 2\cdot 1 + 2 + C = 5$$

$$\boxed{c = 3/4}$$

$$f(4) = \frac{4^4}{4} + \frac{2}{4} + 2\cdot 4 + 3/4 = 64 + \frac{1}{2} + 8 + \frac{3}{4}$$

$$= 73.25$$

10

$$y' = 2x e^y \qquad y(2)=0 \qquad y(1)=?$$

$$\frac{y'}{e^y} = 2x \implies \boxed{e^{-y}y' = 2x}$$

$$\int \underbrace{e^{-y}\,\overbrace{y'\,dx}^{dy}} = \int 2x\,dx$$

$$\boxed{\ln a^b = b\ln a}$$

$$-e^{-y} + C = x^2$$

$$C - x^2 = e^{-y} \qquad \ln(c - x^2) = -y$$

$$\boxed{y = -\ln(c - x^2)}$$

$$0 = -\ln(c-4)$$

$$c - 4 = 1 \qquad \boxed{c = 5}$$

$$y(1) = -\ln(5 - 1^2) = -\ln(4) = -2\ln 2$$

$$-\ln(2^2)$$

(10) $\lim_{x\to 0} \frac{1}{x} \int_0^{2x} \frac{\sin t}{t}\, dt$

$$= 2\lim_{x\to 0} \frac{\int_0^{2x} \left(\frac{\sin t}{t}\right) dt}{2x} \quad \begin{matrix}\to 0 \\ \\ \searrow 0\end{matrix}$$

$$= 2 \lim_{u\to 0} \frac{\int_0^{u} \left(\frac{\sin t}{t}\right) dt}{u} \quad \begin{matrix}\to 0 \\ \\ \searrow 0\end{matrix}$$

$$\underset{\text{L'Hospital}}{=} 2 \lim_{u\to 0} \frac{\left(\frac{\sin u}{u}\right)}{1} = 2\cdot 1 = 2$$

$$\frac{d}{du} \int_0^{u} g(t)\, dt = g(u)$$

$$\frac{d}{du} \underbrace{\int_0^{2u} g(t)}$$

$$\frac{d}{du} G(2u) = G'(2u)\cdot 2$$