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(4) $F(x)$ fonksiyonu $f(x)$'in bir anti-türevi ve $F(-4)=2$ ve $F(-8)=8$ ise

$$\int_1^2 f(-4x)\,dx \text{ nedir?}$$

Çözüm değişken değişimi yapalım.

$u = -4x$ olsun. $du = (-4)dx \Rightarrow dx = -\frac{1}{4}du$

$x=1 \Rightarrow u=-4$

$x=2 \Rightarrow u=-8$

$$\int_1^2 f(-4x)\,dx = \int_{-4}^{-8} f(u)\left(-\frac{1}{4}\right)du = -\frac{1}{4}\int_{-4}^{-8} f(u)\,du$$

$$= -\frac{1}{4}\big(F(-8)-F(-4)\big) = -\frac{1}{4}(8-2) = -3/2$$

(6) $\lim\limits_{x\to 0} \dfrac{2-e^{2x}-e^{-2x}}{x^2} = \dfrac{0}{0}$ olduğu için L'Hospital metodunu uygulamamız gerekiyor

$$\underset{LH}{=} \lim_{x\to 0} \frac{0-e^{2x}(2)-e^{-2x}(-2)}{2x}$$

$$= \lim_{x\to 0} \frac{-\cancel{2}\left(e^{2x}-e^{-2x}\right)}{\cancel{2}x} = \frac{0}{0}$$ yeniden L'Hospital

$$\underset{LH}{=} \lim_{x\to 0} \frac{-\left(e^{2x}(2)-e^{-2x}(-2)\right)}{1} = \frac{-(2+2)}{1} = -4$$

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(8) $\int_1^4 e^{\sqrt{x}}\,dx$ değişken dönüşümü

$z=\sqrt{x} \qquad dz = \underbrace{\frac{1}{2\sqrt{x}}}_{2z} dx$

$x=1 \to u=1$
$x=4 \to u=2$

$\boxed{2z\,dz = dx}$

$= \int_1^2 e^z\, 2z\,dz = 2\int_1^2 z e^z\,dz$

$u=z \qquad dv = e^z dz$
$du = dz \qquad v = e^z$

$\int u\,dv = uv - \int v\,du$

$= 2\left(ze^z\Big|_1^2 - \int_1^2 e^z\,dz\right)$

$= 2(ze^z - e^z)\Big|_1^2 = 2(z-1)e^z\Big|_1^2$

$= 2e^2$

(10) $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)} = \sum_{n=1}^{\infty} \frac{A}{3n-2} + \frac{B}{3n+1}$

$0n+1 = A(3n+1) + B(3n-2)$

$= 3(A+B)n + (A-2B)$

$A+B=0 \Rightarrow B=-A \Rightarrow B=-1/3$

$A-2B=1 \Rightarrow 3A=1 \Rightarrow A=1/3$

$= \sum_{n=1}^{\infty} \frac{1/3}{3n-2} - \frac{1/3}{3n+1}$

$= \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{3n-2} - \frac{1}{3n+1} = \lim_{N\to\infty} \frac{1}{3}\sum_{n=1}^{N} \frac{1}{3n-2} - \frac{1}{3n+1}$

$= \lim_{N\to\infty} \frac{1}{3}\left(\frac{1}{1} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cdots + \cancel{\frac{1}{3N-2}} - \frac{1}{3N+1}\right)$

$= \lim_{N\to\infty} \frac{1}{3}\left(1 - \frac{1}{3N+1}\right) = \frac{1}{3}$

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(16) $f(x) = x^x$ ise $f'(1) = ?$

$$f(x) = e^{x\ln x} \qquad \left[\, a^b = e^{\ln a^b} = e^{b\ln a} \,\right]$$

$$f'(x) \underset{\uparrow}{=} \underbrace{e^{x\ln x}}\;\underbrace{\left(1\cdot \ln x + x\cdot \frac{1}{x}\right)}$$

zincir kural

çarpım kuralı $\frac{d}{dx}(x\ln x)$

$$f'(x) = x^x(\ln x + 1)$$

$$f'(1) = 1^1(\underbrace{\ln 1}_{0} + 1) = 1$$

(22) $f(x) = \begin{cases} 1 & -2 \le x \le 0 \\ \dfrac{1}{\sqrt{16-x^2}} & 0 < x \le 4 \end{cases}$

$$\int_{-2}^{4} f(x)\,dx = \underbrace{\int_{-2}^{0} 1\,dx}_{A} + \underbrace{\int_{0}^{4} \frac{1}{\sqrt{16-x^2}}\,dx}_{B}$$

$$A = \int_{-2}^{0} 1\,dx = x\Big|_{-2}^{0} = (0-(-2)) = 2$$

$$B = \int_{0}^{4} \frac{1}{\sqrt{16-x^2}}\,dx \qquad x = 4\sin\theta \quad dx = 4\cos\theta\,d\theta$$

$x = 0 \Rightarrow \theta = 0$

$x = 4 \Rightarrow \theta = \pi/2$

devamı arkada

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$$B = \int_0^{\pi/2} \frac{4\cos\theta}{\sqrt{16-16\sin^2\theta}}\, d\theta = 4\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{16(1-\sin^2\theta)}}\, d\theta$$

$$= \frac{4}{4}\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{\cos^2\theta}}\, d\theta = \int_0^{\pi/2} \frac{\cos\theta}{\cos\theta}\, d\theta$$

0 ve $\pi/2$ arasında
$\cos\theta \geq 0$ o yüzden
$|\cos\theta|$ değil $\cos\theta$ yazabiliriz

$$= \int_0^{\pi/2} d\theta = \theta \Big|_0^{\pi/2} = \pi/2$$

$$\int_{-2}^{4} f(x)\,dx = A + B = 2 + \pi/2$$

24) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n = ?$

$$f(x) = \sum_{n=1}^{\infty} n x^n = x\sum_{n=1}^{\infty} n x^{n-1}$$

$$= x \cdot \frac{d}{dx}\sum_{n=0}^{\infty} x^n = x \cdot \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\boxed{f(x) = x \cdot \frac{1}{(1-x)^2}}$$ Aradığımız seri $f\left(\frac{2}{3}\right)$

$$f\left(\frac{2}{3}\right) = \frac{2}{3}\cdot\frac{1}{\left(1-\frac{2}{3}\right)^2} = \frac{2}{3}\cdot\frac{1}{\left(\frac{1}{9}\right)} = \frac{2}{3}\cdot\frac{9}{1}$$

$$= 6 //$$

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